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SUNYAEV-ZEL'DOVICH EFFECT BY MULTIPLE SCATTERING : NUMERICAL SOLUTION OF THE TRANSFER EQUATIONS

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The radiative transfer equations for multiple inverse Compton scattering of the Cosmic Microwave Background Radiation (CMBR) by the hot intra-cluster electrons are solved numerically. The spherical isothermal and inhomogeneous β model has been considered for the electron distribution. The anisotropy of the CMBR caused by scattering, known as thermal Sunyaev-Zel'dovich effect, along the radial axis of the medium is compared with the analytical solution of Kompaneets equation. The X-ray data of several clusters of galaxies at low redshifts provide an estimation of the central electron density n_0 to be of the order 10^{-3}. It is found that for this value of n_0 the effect of multiple scattering is negligible. The numerically calculated anisotropy along the radial axis matches well with the analytical solution that describes single scattering. The result incorporating multiple scattering is fitted with the recent observation of Sunyaev-Zel'dovich effect in the cluster Abell 2163. It is shown that if n_0 is greater by an order of magnitude, which could be possible for cluster of galaxies at comparatively higher redshift, multiple scattering would play a significant role at the Wien region of the anisotropy spectrum. A fitting formula for the correction to the Sunyaev-Zel'dovich effect due to multiple scattering is provided.

Keywords: Cosmology - radiative transfer - scattering - galaxies: cluster

1. Introduction

Inverse Compton scattering of the Cosmic Microwave Background Radiation (CMBR) by hot intra-cluster gas - better known as the thermal Sunyaev-Zel'dovich effect (TSZE)[1−3] results in a systematic transfer of photons from the Rayleigh-Jeans to the Wien side of the spectrum causing a distortion in the Planckian nature of the spectrum. The measurements of the effect yield directly the properties of the hot intra-cluster gas, the total dynamical mass of the cluster as well as the indirect information on the cosmological evolution of the clusters. The effect is also used as an important tool to determine the Hubble constant H_0 and the density parameter Ω_0 of the Universe[4−10]. Recent interferometric imaging of the TSZE[11−14] has been used to estimate the mass of various galaxy clusters which could constrain the cosmological parameters of structure formation models. Measurements of TSZE and the kinetic Sunyaev Zel'dovich effect (KSZE) from the Sunyaev-Zel'dovich Infrared Experiment and OVRO/BIMA determine the central Compton y parameter and

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constrain the radial peculiar velocity of the clusters. In near future sensitive observations of the effect with ground based and balloon-borne telescopes, equipped with bolometric multi-frequency arrays, are expected to yield high quality measurements.

In modeling the observational data, the analytical solution of Kompaneets equation[15] with relativistic corrections[16] are used in general. All theoretical discussions and observational inferences so far are restricted to the case of single Compton scattering. Single scattering approximation is well justified because X-ray data of most of the galaxy clusters wherein the Sunyaev-Zel'dovich effect has been investigated, indicate low optical depth of the intra-cluster medium (ICM). Effects of multiple scattering have been considered by many authors[17−21]. Recently, Itoh, Kawana, Nozawa and Kohyama[22] have studied the effect of multiple scattering by performing direct numerical integration of the collision term of the Boltzmann equation and found that the effect of multiple scattering is negligible for the observed galaxy clusters.

In this paper, the numerical solutions of the most general radiative transfer equation for inverse Compton scattering of CMBR photon are presented by incorporating multiple scattering in a spherically symmetric inhomogeneous and isothermal medium. Although my approach is different, the results are in good agreement with that of Itoh et al.[22]. I further present the results by considering a medium with optical depth an order of magnitude higher than that of the galaxy clusters observed so far.

In section 2, I present the most general radiative transfer equation that describes the inverse Compton multiple scattering of low energy photons with non-relativistic electrons in an inhomogeneous medium. The Kompaneets equation which is a special case of isotropic scattering is also presented with the analytical solution that describes the anisotropy in the CMBR spectrum due to single scattering. The model parameters are provided in section 3. The results are discussed in section 4 followed by the conclusion in section 5.

2. The Radiative Transfer Equations

2.1. *General case*

Chandrasekhar[23] has provided the radiative transfer equations for the Compton scattering by assuming that the electron energy is much less than the photon energy. On the other hand, for the scattering of CMBR, the photon energy is much less than the electron energy. In fact, in the TSZE, the electrons are considered to have relativistic motion described by relativistic Maxwellian distribution. The relevant radiative transfer equation in its general form that describes the inverse Compton multiple scattering of low energy photons in a spherically symmetric inhomogeneous medium can be written as[24−26]

$$\frac{\partial I_\nu}{\partial s} = \mu \frac{\partial I_\nu(\mu, r)}{\partial r} + \frac{1 - \mu^2}{r} \frac{\partial I_\nu(\mu, r)}{\partial \mu}$$

$$= -\sigma_T n_e(r) I_\nu(\mu, r) + \frac{1}{2}\sigma_T n_e(r)\omega_0 \times$$

$$\int_{-1}^{1} \left\{ P_0 - \frac{2kT_e}{m_e c^2}P_1 + \frac{2kT_e}{m_e c^2}\left(\nu^2\frac{\partial^2}{\partial\nu^2} - 2\nu\frac{\partial}{\partial\nu}\right)P_2 \right\} I_\nu(\mu, r)d\mu'. \quad (1)$$

The above equation describes the inverse Compton scattering of low-frequency radiation (first order in $h\nu/mc^2$) in a hot thermal isotropic electron gas (first order in kT_e/mc^2) in the rest frame of the electron gas. Here ω_0 is the albedo for single scattering, s is the ray path, $\mu = \cos\theta$ where θ is the angle between the axis of symmetry (radial axis) in the rest frame of electron gas and the ray path, σ_T and n_e being the Thomson scattering cross section and the electron number density respectively. k, c, m_e, ν and T_e are Boltzmann constant, velocity of light, electron rest mass, frequency of the photon and the temperature of the electron respectively. $\omega_0 = 1$ for a purely scattering medium. The phase functions P_0, P_1 and P_2 are given as :

$$P_0(\mu, \mu') = \frac{3}{8}\left[3 - \mu^2 - \mu'^2(1 - 3\mu^2)\right], \quad (2)$$

$$P_1(\mu, \mu') = \frac{3}{8}\left[1 - 3\mu'^2 - 3\mu^2(1 - 3\mu'^2) + 2\mu^3\mu'(3 - 5\mu'^2) + 2\mu\mu'(3\mu'^2 - 1)\right], \quad (3)$$

and

$$P_2(\mu, \mu') = \frac{3}{8}\left[3 - \mu^2 - \mu'^2 + \mu\mu'(3\mu\mu' - 5 + 3\mu^2 + 3\mu'^2 - 5\mu^2\mu'^2)\right]. \quad (4)$$

2.2. *Special case : Isotropic radiation field*

For an isotropic radiation field

$$\frac{1}{2}\int_{-1}^{1} I(\mu', \nu)d\mu' = I(\nu), \quad (5)$$

$$\frac{1}{2}\int_{-1}^{1} I(\mu', \nu)\mu'^2 d\mu' = \frac{1}{3}I(\nu) \quad (6)$$

and

$$\int_{-1}^{1} I(\mu', \nu)\mu' d\mu' = \int_{-1}^{1} I(\mu, \nu)\mu'^3 d\mu' = 0. \quad (7)$$

Therefore, for isotropic scattering, equation (1) reduces to

$$\frac{\partial I_\nu}{\partial s} = \sigma_T n_e(r)\frac{2kT_e}{m_e c^2}\left(\nu^2\frac{\partial^2 I_\nu}{\partial\nu^2} - 2\nu\frac{\partial I_\nu}{\partial\nu}\right) \quad (8)$$

which is well known as the Kompaneets equation for low frequency photons.

Integration of equation (8) along the ray path provides the thermal component of the distortion ΔI and is written as

$$\Delta I(x) = I_0 y\frac{x^4 e^x}{(e^x - 1)^2}\left[\frac{x(e^x + 1)}{e^x - 1} - 4\right](1 + \delta_T) \quad (9)$$

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where $x = \frac{h\nu}{kT_{CMBR}}$, $I_0 = 2(kT_{CMBR})^3/(hc)^2$. The term $y = \frac{2kT_e}{m_e c^2}\int \sigma_T n_e ds$ is usually referred to as the Compton y parameter and δ_T is a relativistic correction to the thermal effect[16] significant if $kT_e > 5$KeV.

3. The Models

I have adopted the isothermal β model[27] for the density distribution of the clusters. The spherical model density is described by

$$n_e(r) = n_0 \left(1 + \frac{r^2}{r_c^2}\right)^{-3\beta/2},\qquad(10)$$

where the core radius r_c and β are shape parameters, n_0 is the central electron density.

I have adopted the values of the model parameters given in LaRoque et al.[13] for the Sunyaev-Zel'dovich anisotropy spectrum observed in Abell 2163. A combination of ROSAT X-ray data and OVRO/BIMA observation of Sunyaev-Zel'dovich effect of Abell 2163 at the cosmological redshift z=0.203 implies the electron temperature $T_e = 12.4$ KeV, the angular core radius of the cluster $\theta_c = 1'.20 \pm 0'.11$ and the shape parameter $\beta = 0.616\pm0.031$. However, for simplicity, I have adopted $\beta = 2/3$ which also provides a good fit to the observed data. Following LaRoque et al.[13] , I have considered a cosmological model with the matter density parameter $\Omega_m = 0.3$ and the cosmological constant corresponding to $\Omega_v = 0.7$. I have taken the Hubble constant $H_0 = 58.0$ Kms^{-1}Mpc^{-1}. With the above parameters, observational fit of the Sunyaev-Zel'dovich effect for Abell 2163 yields the central electron number density $n_0 = 6.96 \times 10^{-3}$ and hence the corresponding Compton y parameter is 3.83×10^{-4}. In order to investigate the effect of multiple scattering in a denser medium I have also considered models with $10 \times n_0$ and $20 \times n_0$ keeping other parameters unaltered.

4. Results and Discussion

Equation (1) is a coupled integro partial differential equation and so it cannot be solved analytically. I have solved it numerically by discretization method. In this method the medium is divided into several shells and the integration is performed over two dimensional grids of angular and radial points. The numerical method is described in detail in Peraiah & Varghese[28]. I have used 11 points Trapezoidal method for angular and frequency integration. The numerical code is thoroughly tested for stability and flux conservation.

For the boundary condition, I have provided equal amount of intensity corresponding to $T_{CMBR} = 2.728$K at optical depth $\tau = 0$ along all directions, i.e.,

$$I(\pm\mu, \nu, \tau = 0) = \frac{2h\nu^3}{c^2}\left(e^{h\nu/kT_{CMBR}} - 1\right)^{-1}.\qquad(11)$$

The relativistic correction $(1 + \sigma_T)$ is included to the emergent intensity.
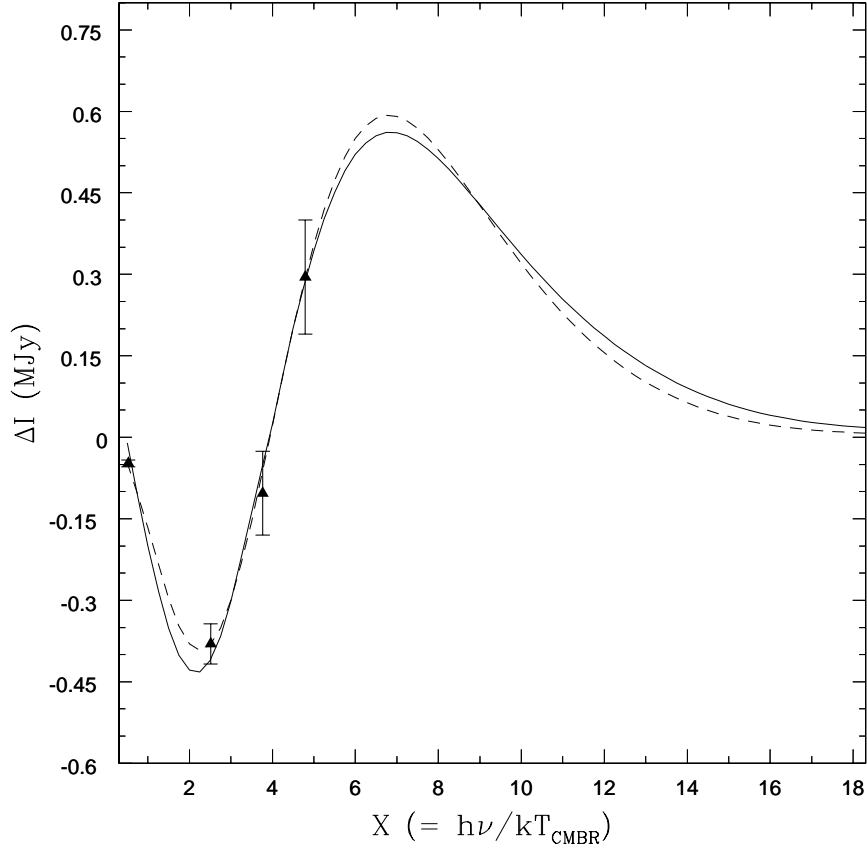
Fig. 1.   Best model fit for thermal Sunyaev-Zel'dovich spectrum of Abell 2163. Solid line represents the distortion along the radial axis for multiple scattering and dashed line represents that along the ray path for single scattering. The central electron number density $n_0 = 6.96 \times 10^{-3}$

Equation (1) decribes the anisotropic and inhomogeneous radiation field in a spherically symmetric medium. Although scattering makes the radiation anisotropic, spherical symmetry of the medium and the initial isotropy of the radiation make the emergent radiation field isotropic. As a consequence the distortion along the radial axis and that along the ray path is the same.

The numerical results that incorporate multiple scattering and the results for single scattering are presented in Fig 1, Fig 2 and Fig 3 for different values of the electron number density. Since the optical depth with a central electron number density $n_0 = 6.96 \times 10^{-3}$ is small, the effect of multiple scattering is negligible as seen in Fig 1. The slight differences in the results at the Wien and Rayleiegh Jeans region is well in agreement with that obtained by Itoh et al.[22] . This can
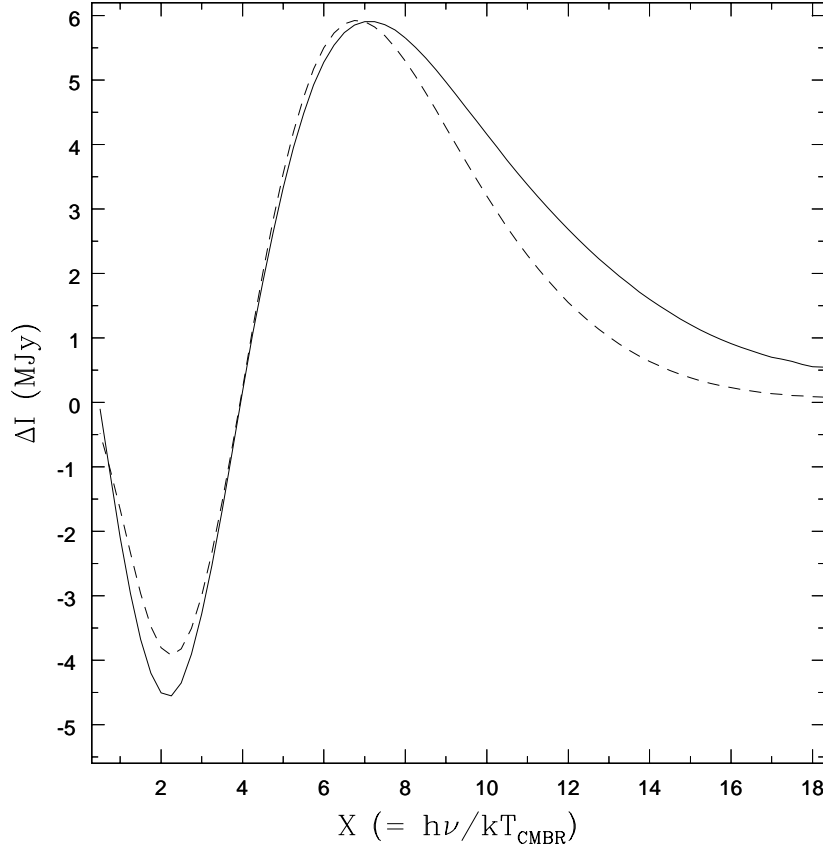
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Fig. 2.   Same as Fig. 1 but with the central electron density $10 \times n_0$.

be visualized from Fig. 1 where I have fitted the observational data for the galaxy cluster Abell 2163. The model fit clearly shows that the effect of multiple scattering can be neglected for Abell 2163 within the uncertainties in the physical parameters.

In Fig. 2 and Fig. 3 the effect of multiple scattering is shown by increasing the central electron density ten and twenty times respectively. The other parameters such as the electron temperature, shape parameters etc. and the cosmological parameters are kept unchanged. With the increase in the electron number density and hence the optical depth of the medium, the number of scattering increases. Consequently, the distortion in the spectrum increases significantly. The increase in the distortion is much more at the Wien region of the spectrum than that at the Rayleigh-Jeans region.

The spectral distortion due to Sunyaev-Zel'dovich effect by single scattering is characterized by three distinct frequencies : the cross over frequency $x_0 = 3.83$
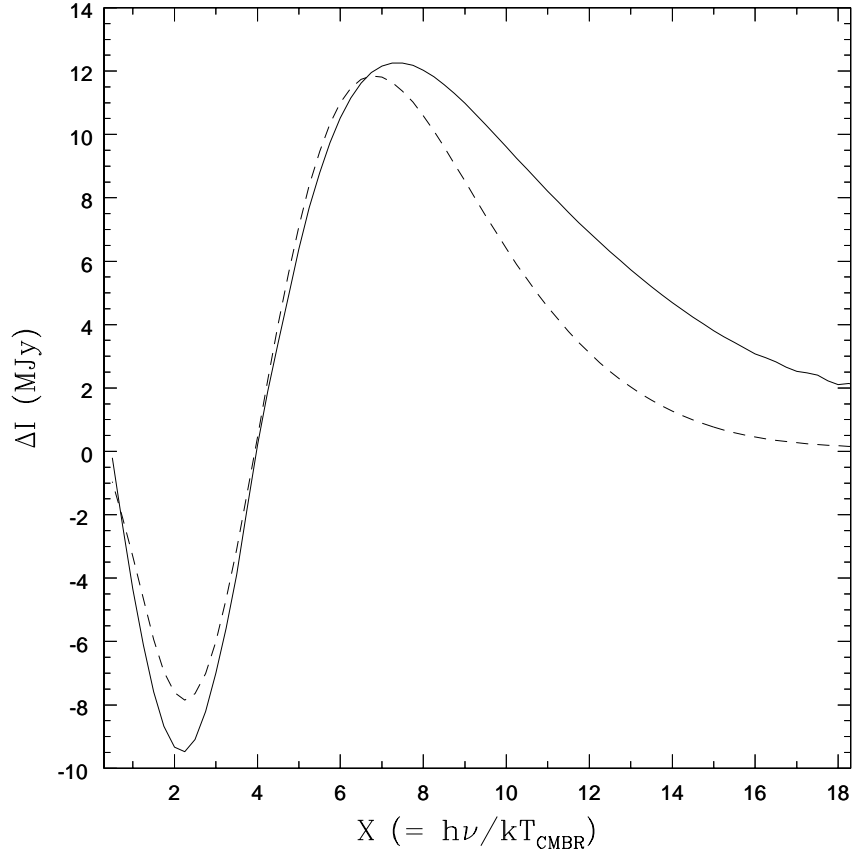
Fig. 3.   Same as Fig. 1 but with the central electron density $20 \times n_0$.

where the thermal Sunyaev Zel'dovich effect vanishes, $x_{min} = 2.26$ which gives the minimum decrement of the CMB intensity and $x_{max} = 6.51$ which gives the maximum distortion. The value of x_0 is pushed to higher values of x with the increase in T_e for the relativistic case. Fig. 2 and Fig. 3 shows that there is no chnage in the values of x_0, x_{min} and a negligible change in x_{max} when multiple scattering is included. The amount of distortion due to multiple scattering is however higher at x_{min} as compared to that with single scattering. At x_{max} the amount of distortion remains almost the same even if the central electron density is increased by twenty times. The effect of multiple scattering becomes significant as x increases from x_{max} that is at the far Wien region. Therefore, multiple scattering must be incorporated in modeling the thermal Sunyaev-Zel'dovich effect anisotropy spectrum if the optical depth of the medium is comparatively high. Quantitatively, if the central electron density of a galaxy cluster is one order more than that of Abell 2163 or other clusters

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observed so far then the effect of multiple scattering would be very important.

Finally, the thermal component of the distortion ΔI modified due to the multiple scattering can be written as :

$$\Delta I(x) = I_0 y \frac{x^4 e^x}{(e^x - 1)^2} \left[\frac{x(e^x + 1)}{e^x - 1} - 4 \right] (1 + \delta_T)(1 + \delta_M) \tag{12}$$

where the correction due to multiple scattering δ_M is expressed by a fitting formula given by

$$\delta_M = 7.2 n_0 \left[1 + \log\left(\frac{0.139}{n_0} \right) \right] (-1.864897 + 2.744115x - 1.337648x^2 + 0.307672x^3$$
$$- 3.812115 \times 10^{-2} x^4 + 2.634191 \times 10^{-3} x^5 - 9.551485 \times 10^{-5} x^6 +$$
$$1.432487 \times 10^{-6} x^7). \tag{13}$$

5. Conclusions

The general transfer equations for inhomogeneous low frequency radiation multiply scattered by relativistic thermal electron gas in a spherically symmetric medium are solved numerically. The observed Sunyaev-Zeldovich effect for Abell 2163 is fitted with the numerical result. The numerical result for the distortion in the CMBR due to multiple scattering is compared with that given by the analytical solution of Kompaneets equation. The effect of multiple scattering is found to be insignificant for the inferred optical depth of the galaxy clusters observed so far. However, if the central electron number density is increased by an order of magnitude, multiple scattering affects the CMBR distortion spectrum significantly in the Wien region. Therefore, it is important to include multiple scattering while modeling the observed Sunyaev-Zel'dovich effect of galaxy clusters with high electron density. A fitting formula for incorporating the correction due to multiple scattering is provided.

Acknowledgments

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